

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

TMA1211 – DISCRETE MATHEMATICS AND PROBABILITY

(All sections / Groups)

1 MARCH 2019
3.00 p.m. – 5.00 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of **6 pages** only excluding the cover page with **5 Questions** and an **Appendix**.
2. Attempt **ALL FIVE (5)** questions.
3. Please write your answers in the Answer Booklet provided, and **start each question on a new page**.
4. Show all steps.
5. Statistical tables and formulas are provided and attached in Appendix.

Answer ALL FIVE (5) questions.

QUESTION 1 (10 marks)

a) Given the arguments as below.

If Edra gets the manager's position and works hard, then he will get a promotion.
If he gets the promotion, he will buy a new house. He has not purchased a new house. Therefore, either Edra did not get the manager's position or he did not work hard.

Let p : Edra gets the manager's position.

q : Edra works hard.

r : Edra gets a promotion.

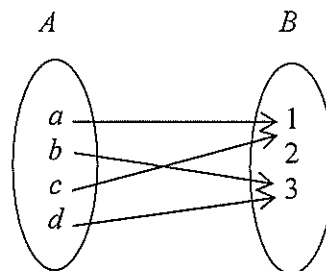
n : Edra buys a new house.

- Write the premises and conclusion in terms of p , q , r and n . (2 marks)
- By using **formal reasoning** (without using a truth table), show that the arguments given are valid. (3 marks)

b) Given sets $A = \{1, 3, 5, 6\}$ and $B = \{2, 6\}$.

- Find the cardinality of power set of B . Then, find power set of B . (2 marks)
- Find $A - B$. (0.5 mark)

c) Let f be a relation as represented in the diagram below.

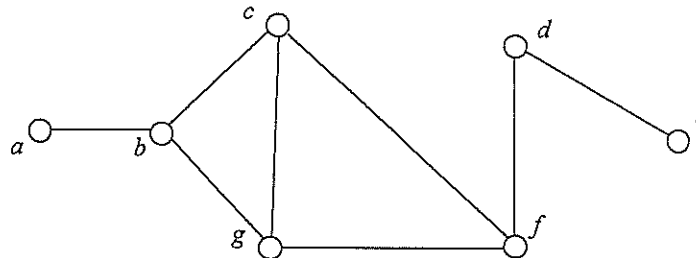


- Determine the domain, co-domain and the range of f . (1.5 marks)
- Is f a function? Why? (1 mark)

Continued.....

QUESTION 2 (10 marks)

a) For the graph below,



- i. find the number of vertices, the number of edges, and the degree of each vertex. (2 marks)
 - ii. verify that Handshaking Lemma is true for this graph. (1.5 marks)
- b) Determine whether the set $P = \{x: x \text{ is even}\}$ is closed under the operations of addition and multiplication. Give your reason. (2 marks)
- c) Let $*$ be the operation on the set R of real numbers defined by $a * b = 3ab - a - b$.
- i. Find $1 * 2$. (1 mark)
 - ii. Is $(R, *)$ associative? (2.5 marks)
 - iii. Is $(R, *)$ commutative? (1 mark)

QUESTION 3 (10 marks)

- a) Suppose that f is defined recursively by $f(0) = 5$ and $f(n+1) = 3f(n) - 8$. Find $f(1)$ and $f(2)$. (2 marks)
- b) Consider the sequence 1, 3, 4, 7, 11, 18, 29, ... Find the recursive definition of the given sequence. [HINT: Start by observing the third term together with its previous two terms.] (3 marks)
- c) The state table of a finite-state machine, $M = (S, I, O, f, g, S_0)$, is given below.

State	f		g	
	Input		Output	
	0	1	0	1
S_0	S_1	S_0	1	0
S_1	S_3	S_0	1	1
S_2	S_1	S_2	0	1
S_3	S_2	S_1	0	0

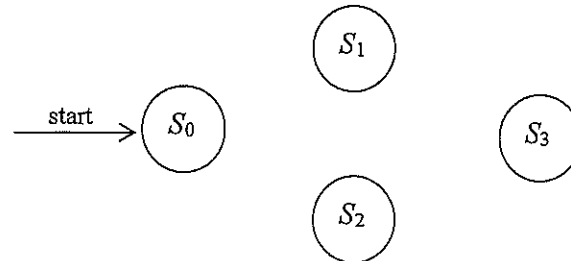
It is known that S_0 is the accepting state.

Continued.....

QUESTION 3c) (Continued)

- i. Copy the **draft** state diagram of M below and complete it on your answer script. Remember to indicate the accepting state on your answer script.

(2 marks)



- ii. Copy the table below and complete it on your answer script. (3 marks)

Input string	Output string	Last State	Accepted? (Yes/ No)
100001			
1001			

QUESTION 4 (10 marks)

- a) Beta Manufacturing Sdn. Bhd. is studying the effects of heating time, heating temperature, and type of glue used in producing light-emitting diodes (LED). Three different heating time, four different temperatures, and three different glues were studied. Find the total number of outcomes to be studied. (1 mark)
- b) There are 6 students who are waiting for the school bus. In how many ways can 6 students be lined up to get on the bus? (1 mark)
- c) Mary goes to the library. The probabilities that she checks out a chemistry book is 0.40, a history book is 0.30, and both chemistry and history books is 0.20. What is the probability that she checks out a chemistry book or history book or both? (2 marks)
- d) Hazardous chemical wastes are usually residues of chemical, manufacturing or mining operations. The following frequency table shows the distribution of 58 landfills in Melaka according to their concentration of the three hazardous chemicals: arsenic (A), barium (B) and mercury (M).

		Barium (B)			
		High (H)		Low (L)	
		Mercury (M)		Mercury (M)	
		High (H)	Low (L)	High (H)	Low (L)
Arsenic (A)	High (H)	1	3	5	9
	Low (L)	4	8	10	18

Continued.....

QUESTION 4d) (Continued)

If a landfill is selected at random, find the probability that it has a

- i. high concentration of mercury. (2 marks)
- ii. high concentration of barium and low concentrations of arsenic and mercury. (2 marks)
- iii. high concentration of any one of the chemicals and low concentrations of the other two. (2 marks)

Question 5 (10 marks)

- a) According to a 2015 finding by the American Society of Safety Professionals based on the survey with safety professionals (including safety directors, safety specialists and safety managers), two out of five workplace hand injuries are from cuts or punctures.

- i. Based on the survey, what is the probability that the hand injuries are from cuts or punctures? (1 mark)

Assume that the result holds true for current year, find the probability that among 15 hand injuries

- ii. 5 are from cuts or punctures. (1.5 marks)
- iii. between 7 and 11 are **NOT** from cuts or punctures. (1.5 marks)

- b) The joint probability distribution of the random variables X and Y is as follows.

$P(x,y)$		y		
x		0	1	2
	1	0.24	0.25	0.08
	2	0.10	0.10	0.23

- i. Find the marginal probability distribution of X . (1.5 marks)
- ii. By using the answer from part (i) or other method, find the expected value of X , $E(X)$. (1 mark)

- c) Given the joint probability density function of X and Y as below.

$$f(x,y) = \begin{cases} kxy & , \quad 0 < x < 1, 0 < y < 4 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

- i. Find the value of k . (2 marks)
- ii. If $E(X) = \frac{2}{3}$, $E(Y) = \frac{8}{3}$ and $E(XY) = \frac{16}{9}$, are X and Y correlated? (1.5 marks)

Continued.....

Appendix

Table 1: Important Equivalences

<u>Identity laws</u> $p \wedge T \Leftrightarrow p$ $p \vee F \Leftrightarrow p$	<u>Negation laws</u> $p \vee \neg p \Leftrightarrow T$ $p \wedge \neg p \Leftrightarrow F$	<u>Others</u> $p \rightarrow q \Leftrightarrow \neg p \vee q$ (CI)
<u>Domination laws</u> $p \vee T \Leftrightarrow T$ $p \wedge F \Leftrightarrow F$	<u>Absorption Laws</u> $p \vee (p \wedge q) \Leftrightarrow p$ $p \wedge (p \vee q) \Leftrightarrow p$	
<u>Idempotent laws</u> $p \vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	<u>De Morgan's laws</u> $\neg(p \wedge q) \Leftrightarrow (\neg p) \vee (\neg q)$ $\neg(p \vee q) \Leftrightarrow (\neg p) \wedge (\neg q)$	
<u>Double Negation laws</u> $\neg(\neg p) \Leftrightarrow p$	<u>Commutative laws</u> $p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	
<u>Associative laws</u> $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$ $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	<u>Distributive laws</u> $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	

Table 2: Rules of Inference

Rules of Inference	Tautology	Name
$\frac{p}{\therefore p \vee q}$	$p \rightarrow (p \vee q)$	Addition
$\frac{p \wedge q}{\therefore p}$	$(p \wedge q) \rightarrow p$	Simplification
$\frac{p}{\therefore p \wedge q}$	$[(p) \wedge (q)] \rightarrow (p \wedge q)$	Conjunction
$\frac{p \quad p \rightarrow q}{\therefore q}$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus Ponens
$\frac{\neg q \quad p \rightarrow q}{\therefore \neg p}$	$[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$	Modus Tollens
$\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothetical Syllogism
$\frac{p \vee q \quad \neg p}{\therefore q}$	$[(p \vee q) \wedge \neg p] \rightarrow q$	Disjunctive Syllogism
$\frac{p \vee q \quad \neg p \vee r}{\therefore q \vee r}$	$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$	Resolution

Continued.....

Formulas

- Handshaking Lemma: $2e = \sum \deg(v)$

Probability

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(C|D) = \frac{P(C \cap D)}{P(D)}$

Joint Discrete Probability Distribution

- Expected value of X , $E(X) = \sum_x x \cdot P(X = x)$

Joint Continuous Probability Distribution

- $\int \int_{y \ x} f(x, y) dx dy = 1$
- $\text{cov}(XY) = E(XY) - E(X)E(Y)$

Statistical Table 1: Cumulative Binomial Probabilities

c. $n = 15$

		p														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
x	0	.860	.463	.206	.035	.013	.005	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.990	.829	.549	.167	.080	.035	.005	.000	.000	.000	.000	.000	.000	.000	.000
	2	1.000	.964	.816	.398	.236	.127	.027	.004	.000	.000	.000	.000	.000	.000	.000
	3	1.000	.995	.944	.648	.461	.297	.091	.018	.002	.000	.000	.000	.000	.000	.000
	4	1.000	.999	.987	.836	.686	.515	.217	.059	.009	.001	.000	.000	.000	.000	.000
	5	1.000	1.000	.998	.939	.852	.722	.402	.151	.034	.004	.001	.000	.000	.000	.000
	6	1.000	1.000	1.000	.982	.943	.869	.610	.304	.095	.015	.004	.001	.000	.000	.000
	7	1.000	1.000	1.000	.996	.983	.950	.787	.500	.213	.050	.017	.004	.000	.000	.000
	8	1.000	1.000	1.000	.999	.996	.985	.905	.696	.390	.131	.057	.018	.000	.000	.000
	9	1.000	1.000	1.000	1.000	.999	.996	.966	.849	.597	.278	.148	.061	.002	.000	.000
	10	1.000	1.000	1.000	1.000	1.000	.999	.991	.941	.783	.485	.314	.164	.013	.001	.000
	11	1.000	1.000	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.539	.352	.056	.005	.000
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.764	.602	.184	.036	.000
	13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.920	.833	.451	.171	.010
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.987	.965	.794	.537	.140

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